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GALILEO AND PLATO

By Alexandre Koyré

The name of Galileo Galilei is indissolubly linked with the scientific revolution of the sixteenth century, one of the profoundest, if not the most profound, revolution of human thought since the invention of the Cosmos by Greek thought: a revolution which implies a radical intellectual "mutation," of which modern physical science is at once the expression and the fruit.[1]

This revolution is sometimes characterized, and at the same time explained, as a kind of spiritual upheaval, an utter transformation of the whole fundamental attitude of the human mind; the active life, the *vita activa* taking the place of the θεωρία, the *vita contemplativa,* which until then had been considered its highest form. Modern man seeks the domination of nature, whereas medieval or ancient man attempted above all its contemplation. The mechanistic trend of classical physics—of the Galilean, Cartesian, Hobbesian physics, *scientia activa, operativa,* which was to render man "master and possessor of nature"—has, therefore, to be explained by this desire to dominate, to act; it has to be considered purely and simply an outflow of this attitude, an application to nature of the categories of thinking of *homo faber.*[2] The science of Descartes—and *a fortiori* that of Galileo—is nothing else than (as has been said) the science of the craftsman or of the engineer.[3]

I must confess that I do not believe this explanation to be entirely correct. It is true, of course, that modern philosophy, as well as modern ethics and modern religion, lays much more stress on action, on πράξις, than ancient and medieval thought. And it is just as true of modern science: I am thinking of the Cartesian physics and its analogies of pulleys, strings and levers. Still the attitude we have just described is much more that of Bacon—whose

[1] Cf. J. H. Randall, Jr., *The Making of the Modern Mind* (Boston, 1926), 220 sq., 231 sq.; cf. also A. N. Whitehead, *Science and the Modern World* (New York, 1925).

[2] This widespread conception must not be confused with that of Bergson, for whom all physics, the Aristotelian just as much as the Newtonian, is in the last analysis the work of *homo faber.*

[3] Cf. L. Labertonnière, *Études sur Descartes* (Paris, 1935), II, 288 sq.; 297; 304: "physique de l'exploitation des choses."

rôle in the history of science is not of the same order[4]—than that of Galileo or Descartes. Their science is made not by engineers or craftsmen, but by men who seldom built or made anything more real than a theory.[5] The new ballistics was made not by artificers and gunners, but against them. And Galileo did not learn *his* business from people who toiled in the arsenals and shipyards of Venice. Quite the contrary: he taught them *theirs*.[6] Moreover, this theory explains too much and too little. It explains the tremendous scientific progress of the seventeenth century by that of technology. And yet the latter was infinitely less conspicuous than the former. Besides, it forgets the technological achievements of the Middle Ages. It neglects the lust for power and wealth which, throughout its history, inspired alchemy.

Other scholars have insisted on the Galilean fight against authority, against tradition, especially against that of Aristotle:

[4] Bacon is the announcer, the *buccinator* of modern science, not one of its creators.

[5] The Cartesian and Galilean science has, of course, been of extreme importance for the engineer and the technician; ultimately it has produced a technical revolution. Yet it was created and developed neither by engineers nor technicians, but by theorists and philosophers.

[6] "Descartes artisan" is the conception of Cartesianism developed by Leroy in his *Descartes social* (Paris, 1931), and brought to absurdity by F. Borkenau in his book *Der Uebergang vom feudalen zum bürgerlichen Weltbild* (Paris, 1934). Borkenau explains the birth of the Cartesian philosophy and science by that of a new form of economic enterprise, i.e., manufacturing. Cf. the criticism of the work of Borkenau, a criticism much more interesting and instructive than the book itself, by H. Grossmann, "Die gesellschaftlichen Grundlagen der mechanistischen Philosophie und die Manufaktur," *Zeitschrift für Sozialforschung* (Paris, 1935).

As for Galileo, he is linked with the traditions of the artisans, builders, engineers, etc., of the Renaissance by L. Olschki, *Galileo und seine Zeit* (Halle, 1927), and more recently by E. Zilsel, "The Sociological Roots of Science," *The American Journal of Sociology,* XLVII (1942). Zilsel stresses the rôle played by the "superior artisans" of the Renaissance in the development of the modern scientific mentality. It is, of course, perfectly true that the artists, engineers, architects, etc., of the Renaissance played an important part in the struggle against the Aristotelian tradition, and that some of them—like Leonardo da Vinci and Benedetti—attempted even to develop a new, anti-Aristotelian dynamics; yet this dynamics, as was conclusively shown by Duhem, was in its main features that of the Parisian nominalists, the *impetus* dynamics of John Buridan and Nicole Oresme. And if Benedetti, by far the most remarkable of these "forerunners" of Galileo, transcends sometimes the level of the "Parisian" dynamics, it is not because of his work as engineer and gunner but because of his study of Archimedes and his decision to apply "mathematical philosophy" to the investigation of nature.

against the scientific and philosophical tradition, upheld by the Church and taught in the universities. They have stressed the rôle of observation and experience in the new science of nature.[7] It is perfectly true, of course, that observation and experimentation form one of the most characteristic features of modern science. It is certain that in the writings of Galileo we find innumerable appeals to observation and to experience, and bitter irony toward men who didn't believe their eyes because what they saw was contrary to the teaching of the authorities, or, even worse, who (like Cremonini) did not want to look through Galileo's telescope for fear of seeing something which would contradict their traditional theories and beliefs. It is obvious that it was just by building a telescope and by looking through it, by careful observation of the moon and the planets, by his discovery of the satellites of Jupiter, that Galileo dealt a crushing blow to the astronomy and the cosmology of his times.

Still one must not forget that observation and experience, in the sense of brute, common-sense experience, did not play a major rôle—or, if it did, it was a negative one, the rôle of obstacle—in the foundation of modern science.[8] The physics of Aristotle, and even more that of the Parisian Nominalists, of Buridan and Nicole Oresme, was, as stated by Tannery and Duhem, much nearer to common sense experience than those of Galileo and Descartes.[9]

[7] Quite recently a friendly critic has reproached me for having neglected this side of Galileo's teaching. (Cf. L. Olschki, "The Scientific Personality of Galileo," *Bulletin of the History of Medicine,* XII [1942].) I must confess I do not believe I have merited this reproach, though I do indeed believe that science is primarily theory and not the gathering of "facts."

[8] É. Meyerson, *Identité et réalité,* 3 ed. (Paris, 1926), 156, shows very convincingly the lack of accord between "experience" and the principles of modern physics.

[9] P. Duhem, *Le Système du Monde* (Paris, 1913), I, 194 sq.: "Cette dynamique, en effet, semble s'adapter si heureusement aux observations courantes qu'elle ne pouvait manquer de s'imposer, tout d'abord, à l'acceptation des premiers qui aient spéculé sur les forces et les mouvements. . . . Pour que les physiciens en viennent à rejeter la Dynamique d'Aristote et à construire la Dynamique moderne, il leur faudra comprendre que les faits dont ils sont chaque jour les témoins ne sont aucunement les faits simples, élémentaires, auxquelles les lois fondamentales de la Dynamique se doivent immédiatement appliquer; que la marche du navire tiré par les haleurs, que le roulement sur une route de la voiture attelée doivent être regardés comme des mouvements d'une extrême complexité; en un mot que pour le principe de la science du mouvement, on doit, par abstraction, considérer un mobile qui, sous l'action d'une force unique, se meut dans le vide. Or, de sa Dynamique Aristote va jusqu'à conclure qu'un tel mouvement est impossible."

It is not "experience," but "experiment," which played—but only later—a great positive rôle. Experimentation is the methodical interrogation of nature, an interrogation which presupposes and implies a *language* in which to formulate the questions, and a dictionary which enables us to read and to interpret the answers. For Galileo, as we know well, it was in curves and circles and triangles, in mathematical or even more precisely, in *geometrical language*—not in the language of common sense or in that of pure symbols—that we must speak to Nature and receive her answers. Yet obviously the choice of the language, the decision to employ it, could not be determined by the experience which its use was to make possible. It had to come from other sources.

Still other historians of science and philosophy[10] have more modestly tried to characterize modern physics, as *physics,* by some of its salient traits: for instance, by the rôle which the principle of *inertia* plays in it. Perfectly right, once more: the principle of *inertia,* in contradistinction to that of the Ancients, holds an outstanding place in classical mechanics. It is its fundamental law of motion; it implicitly pervades Galilean physics and quite explicitly that of Descartes and of Newton. But this characteristic seems to me to be somewhat superficial. In my opinion it is not enough simply to state the fact. We have to understand and to explain it—to explain why *modern* physics was able to adopt this principle; to understand why, and how, the principle of inertial motion, which to us appears so simple, so clear, so plausible and even self-evident, acquired this status of self-evidence and *a priori* truth whereas for the Greeks as well as for the thinkers of the Middle Ages the idea that a body once put in motion will continue to move forever, appeared as obviously and evidently false, and even absurd.[11]

I shall not try to explain here the reasons and causes that produced the spiritual revolution of the sixteenth century. It is for our purpose sufficient to describe it, to describe the mental or intellectual attitude of modern science by two (connected) characteristics. They are: 1) the destruction of the Cosmos, and therefore

[10] Cf. Kurd Lasswitz, *Geschichte der Atomistik* (Hamburg und Leipzig, 1890), II, 23 sq.; E. Mach, *Die Mechanik in ihrer Entwicklung,* 8 ed. (Leipzig, 1921), 117 sq.; E. Wohlwill, "Die Entdeckung des Beharrunggesetzes," *Zeitschrift für Völkerpsychologie und Sprachwissenschaft,* vols. XIV and XV (1883 and 1884), and E. Cassirer, *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit,* 2 ed. (Berlin, 1911), I, 394 sq.

[11] Cf. É. Meyerson, *op. cit.,* 124 sq.

the disappearance in science of all considerations based on that notion;[12] 2) the geometrization of space—that is, the substitution of the homogeneous and abstract space of Euclidian geometry for the qualitatively differentiated and concrete world-space conception of the pre-galilean physics. These two characteristics may be summed up and expressed as follows: the mathematization (geometrization) of nature and, therefore, the mathematization (geometrization) of science.

The dissolution of the Cosmos means the destruction of the idea of a hierarchically-ordered finite world-structure, of the idea of a qualitatively and ontologically differentiated world, and its replacement by that of an open, indefinite and even infinite universe, united and governed by the same universal laws; a universe in which, in contradiction to the traditional conception with its distinction and opposition of the two worlds of Heaven and of Earth, all things are on the same level of Being. The laws of Heaven and the laws of Earth are merged together. Astronomy and physics become interdependent, and even unified and united.[13] And this implies the disappearance from the scientific outlook of all considerations based on value, on perfection, on harmony, on meaning and on purpose.[14] They disappear in the infinite space of the new Universe. It is in this new Universe, in this new world of a geometry made real, that the laws of classical physics are valid and find their application.

The dissolution of the Cosmos—I repeat what I have already said: this seems to me to be the most profound revolution achieved or suffered by the human mind since the invention of the Cosmos by the Greeks. It is a revolution so profound and so far-reaching that mankind—with very few exceptions, of whom Pascal was one

[12] The *term* remains, of course, and Newton still speaks of the Cosmos and its order (as he speaks of *impetus*), but in an entirely new meaning.

[13] As I have endeavoured to show elsewhere (*Études Galiléennes,* III, *Galilée et la loi d'inertie* [Paris, 1940]) modern science results from this unification of astronomy and physics which enables it to apply the methods of mathematical investigation, till then employed in the study of celestial phenomena, to the study of the phenomena of the sublunar world.

[14] Cf. É. Bréhier, *Histoire de la philosophie,* t. II, fasc. 1 (Paris, 1929), 95: "Descartes dégage la physique de la hantise du Cosmos hellénique, c'est-à-dire de l'image d'un certain état privilégié des choses qui satisfait nos besoins esthétiques. . . . Il n'y a pas d'état privilégié puisque tous les états sont équivalents. Il n'y a donc aucune place en physique pour la recherche des causes finales et la considération du meilleur."

—for centuries did not grasp its bearing and its meaning; which, even now, is often misvalued and misunderstood.

Therefore what the founders of modern science, among them Galileo, had to do, was not to criticize and to combat certain faulty theories, and to correct or to replace them by better ones. They had to do something quite different. They had to destroy one world and to replace it by another. They had to reshape the framework of our intellect itself, to restate and to reform its concepts, to evolve a new approach to Being, a new concept of knowledge, a new concept of science—and even to replace a pretty natural approach, that of common sense, by another which is not natural at all.[15]

This explains why the discovery of things, of laws, which today appear so simple and so easy as to be taught to children—the laws of motion, the law of falling bodies—required such a long, strenuous, and often unsuccessful effort of some of the greatest geniuses of mankind, a Galileo, a Descartes.[16] This fact in turn seems to me to disprove the modern attempt to minimize, or even to deny, the originality, or at least the revolutionary character, of Galileo's thinking; and to make clear that the apparent continuity in the development of medieval and modern physics (a continuity so emphatically stressed by Caverni and Duhem)[17] is an illusion. It

[15] Cf. P. Tannery, "Galilée et les principes de la dynamique," in *Mémoires scientifiques,* VI (Paris, 1926), 399: "Si pour juger le système dynamique d'Aristote, on fait abstraction des préjugés qui dérivent de notre éducation moderne, si on cherche à se replacer dans l'état d'esprit que pouvait avoir un penseur indépendant au commencement du XVIIe siècle, il est difficile de méconnaître que ce système est beaucoup plus conforme que le nôtre à l'observation immédiate des faits."

[16] Cf. my *Études Galiléennes,* II, *La loi de la chute des corps* (Paris, 1940).

[17] Cf. Caverni, *Storia del metodo sperimentale in Italia,* 5 vols. (Firenze, 1891–96), particularly vols. IV and V; P. Duhem, *Le mouvement absolu et le mouvement relatif* (Paris, 1905); "De l'accélération produite par une force constante," *Congrès International de l'histoire des sciences, IIIe session,* (Geneva, 1906); *Études sur Léonard de Vinci: Ceux qu'il a lu et ceux qui l'ont lu,* 3 vols. (Paris, 1909–1913), particularly vol. III, *Les précurseurs parisiens de Galilée.* Quite recently the thesis of continuity has been upheld by J. H. Randall, Jr., in his brilliant article "Scientific Method in the School of Padua," *Journal of the History of Ideas,* I (1940); Randall convincingly shows the progressive elaboration of the method of "resolution and composition" in the teaching of the great logicians of the Renaissance. Yet Randall himself states that there was "one element lacking in Zabarella's formulation of method: he did not insist that the principles of natural science be mathematical" (p. 204), and that Cremonini's *Tractatus de paedia* "sounds like the solemn warning of the great tradition of Aristotelian

is true, of course, that an unbroken tradition leads from the works of the Parisian Nominalists to those of Benedetti, Bruno, Galileo and Descartes. (I myself have added a link to the history of that tradition[18]). Still the conclusion drawn therefrom by Duhem is a delusion: a well-prepared revolution is nevertheless a revolution, and in spite of the fact that Galileo himself in his youth (as well as at times Descartes) shared the views and taught the theories of the medieval critics of Aristotle, modern science, the science born from his efforts and discoveries, *does not* follow the inspiration of the "Parisian forerunners of Galileo"; it places itself at once on a quite different level—on a level which I should like to call the Archimedian one. The true forerunner of modern physics is neither Buridan, nor Nicole Oresme, nor even John Philoponos, but Archimedes.[19]

I

The history of the scientific thought of the Middle Ages and of the Renaissance, now beginning to be somewhat better known,[20] can be divided into two periods. Or better, as the chronological order corresponds only very roughly to that division, the history of scientific thought may be, *grosso modo,* divided into three stages or epochs, which correspond in turn to three different types of thinking: the Aristotelian physics first; then the physics of *impetus,* inaugurated, like everything else, by the Greeks, and elaborated in the current of the Fourteenth century by the Parisian nominalists; and finally modern, mathematical, Archimedian or Galilean physics.

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rational empiricism to the triumphant mathematicians" (*ibid.*). As a matter of fact, it is just this "mathematical emphasis added to the logical methodology of Zabarella" (p. 205) which forms, in my opinion, the content of the scientific revolution of the seventeenth century; and, in the opinion of the time, the dividing line between the followers of Plato and those of Aristotle.

[18] Cf. *Études Galiléennes,* I, *A l'aube de la science classique* (Paris, 1940).

[19] The sixteenth century, at least its latter half, is the period of the reception of the study and of the gradual understanding of Archimedes.

[20] We owe that knowledge chiefly to the works of P. Duhem (to the works cited above, n. 17, must be added: *Les Origines de la statique,* 2 vols. [Paris, 1905], and *Le Système du monde,* 5 vols. [Paris, 1913–17]) and to those of Lynn Thorndike (cf. his monumental *History of Magic and Experimental Science,* 6 vols. [New York, 1923–41]). Cf. equally F. J. Dijksterhuis, *Wal en Worp* (Groningen, 1924).

It is these stages that we find represented in the works of the young Galileo, which thus not only give us information on the history—or the prehistory—of his thought, on the *mobiles* and motives which dominated and inspired it, but present us at the same time, condensed and as it were clarified by the admirable mind of its author, a striking and deeply instructive picture of the whole history of pre-galilean physics. Let us briefly follow this story, beginning with Aristotelian physics.

Aristotelian physics is false, of course; and utterly obsolete. Nevertheless, it is a ''physics,'' that is, a highly though non-mathematically elaborated[21] science. It is not a childish phantasy, nor a brute and verbal restatement of common sense, but a theory, that is, a doctrine which, starting of course with the data of common sense, subjects them to an extremely coherent and systematic treatment.[22]

The facts or data which serve as a basis for this theoretical elaboration are very simple, and in practice we admit them just as did Aristotle. It still seems to all of us ''natural'' to see a heavy body fall ''down.'' And just like Aristotle or St. Thomas, we should be deeply astonished to see a ponderous body—a stone or a bull—rise freely in the air. This would seem to us pretty ''unnatural''; and we would look for an explanation in the action of some hidden mechanism.

In the same way we still find it ''natural'' that the flame of a match points ''up,'' and that we place our pots and pans ''on'' the fire. We should be astonished and should seek for an explanation if, for instance, we saw the flame turn about and point ''down.'' Shall we call this conception, or rather this attitude, childish and simple? Perhaps. We can even point out that according to Aristotle himself science begins precisely by looking for an explanation for things that appear natural. Still, when thermodynamics asserts as a principle that ''heat'' passes from a hot to a cold body, but not from the cold to a hot one, does it not simply translate an intuition of common sense that a ''hot'' body ''naturally'' becomes cold, but that a cold one does not ''naturally'' become hot? And even when we are stating that the center of gravity of a system

[21] The Aristotelian physics is essentially non-mathematical. To present it, as Duhem does, *De l'accélération produite par une force constante,* p. 859, simply as based upon another mathematical formula than ours, is an error.

[22] The systematic character of Aristotelian physics is often not sufficiently appreciated by the modern historian of scientific thought.

tends to take the lowest position and does not rise by itself, are we not simply translating an intuition of common sense, the self-same intuition which Aristotelian physics expresses by its distinction of movement into "natural" and "violent"?[23]

Moreover, Aristotelian physics no more rests content than thermodynamics with merely expressing in its language the "fact" of common sense just mentioned; it transposes it, and the distinction between "natural" and "violent" movements takes its place in a general conception of physical reality, a conception of which the principal features seem to be: (a) the belief in the existence of qualitatively determined "natures," and (b) the belief in the existence of a Cosmos—that is, the belief in the existence of principles of order in virtue of which the entirety of real beings form a hierarchically-ordered whole.

Whole, cosmic order, and harmony: these concepts imply that in the Universe things are (or should be) distributed and disposed in a certain determined order; that their location is not a matter of indifference (neither for them, nor for the Universe); that on the contrary each thing has, according to its nature, a determined "place" in the Universe, which is in some sense its own.[24] A place for everything, and everything in its place: the concept of "natural place" expresses this theoretical demand of Aristotelian physics.

The conception of "natural place" is based on a purely static conception of order. Indeed, if everything were "in order," everything would be in its natural place, and, of course, would remain and stay there forever. Why should it depart from it? On the contrary, it would offer a resistance to any attempt to expel it therefrom. This expulsion could be effected only by exerting some kind of *violence,* and the body would seek to come back, if, and when, owing to such a *violence,* it found itself out of "its" place.

Thus every movement implies some kind of cosmic disorder, a disturbance of the world-equilibrium, being either a direct effect of *violence,* or, on the contrary, the effect of the effort of Being to compensate for the *violence,* to recover its lost and troubled order and balance, to bring things back to their natural places, places where they can rest and remain. It is this returning to order

[23] Cf. E. Mach, *Die Mechanik,* 124 sq.

[24] It is only in "its" place that a being comes to its accomplishment and becomes truly itself. And that is the reason why it tends to reach that place.

which constitutes precisely what we have called "natural" movement.[25]

Upsetting equilibrium, returning to order: it is perfectly clear that order constitutes a firm and durable state which tends to extend itself indefinitely. There is therefore no need to explain the state of rest, at least the state of a body at rest in its natural, proper place; it is its own nature which explains it, which explains, for instance, the earth's being at rest in the center of the world. It is obvious likewise that movement is necessarily a transitory state: natural movement ends naturally when it reaches its goal. And as for violent movement, Aristotle is too optimistic to admit that this abnormal status could endure; moreover, violent movement is disorder creating disorder, and to admit that it could endure indefinitely would mean, in fact, to abandon the very idea of a well-ordered Cosmos. Aristotle therefore holds the reassuring belief that nothing which is *contra naturam possit esse perpetuum.*[26]

Thus, as we have just said, in the Aristotelian physics movement is an essentially transitory state. Taken literally, however, this statement would be incorrect, and even doubly incorrect. As a matter of fact movement, though it is for *each of the moved bodies,* or at least for those of the sublunar world, for the movable things of our experience, a necessarily transitory and ephemeral state, is nevertheless for the whole of the world a necessarily eternal, and therefore an eternally necessary phenomenon[27]—a phenomenon which we cannot explain without discovering its origin and cause in the physical as well as the metaphysical structure of the Cosmos. Such an analysis would show that the ontological structure of material Being prevents it from reaching the state of perfection implied in the notion of absolute rest, and would enable us to see the ultimate physical cause of the temporary, ephemeral and variable movements of sublunar bodies in the continuous, uniform, and perpetual movement of the heavenly spheres.[28] On the other hand, movement strictly speaking is not a *state:* it is a

[25] The conceptions of "natural places" and "natural motions" imply that of a finite Universe.

[26] Aristotle, *Physics,* VIII, 8, 215 b.

[27] Movement can result only from a previous movement. Therefore every actual motion implies an infinite series of preceding ones.

[28] In a finite Universe the only uniform movement which can persist indefinitely is a circular one.

process, a flux, a *becoming,* in and by which things constitute, actualize and accomplish themselves.[29] It is perfectly true that becoming has Being as its end; and that movement has rest as its goal. Yet this immutable rest of a fully actualized being is something utterly different from the heavy and impotent immobility of a being unable to move itself; the first is something positive, is "perfection and *actus*"; the second is only a "privation." Movement, therefore—a *processus,* a becoming, a change—finds itself placed ontologically between the two. It is the being of everything that changes, of which the being is alteration and modification and which *is* only in changing and in modifying itself. The famous Aristotelian definition of movement—*actus entis in potentia in quantum est in potentia*—which Descartes will find perfectly unintelligible—expresses admirably the fact: movement is the being—or the *actus*—of everything which is not God.

To move is thus to change, *aliter et aliter se habere,* to change in itself and in respect to others. This implies on the one hand a term of relation or of comparison, in respect to which the thing moved changes its being or relation; which implies—if we are dealing with local movement[30]—the existence of a fixed point in respect to which the moved moves itself, a fixed unmovable point; which obviously can only be the center of the Universe. On the other hand the fact that every change, every process needs a cause to explain it, implies that every movement needs a mover to produce it, which, as long as the movement endures, keeps it going. Movement indeed does not maintain itself, as rest does. Rest—a state or a privation—does not need the action of any cause to explain its persistence. Movement, change, any process of actualization (or of decay), and even of continuous actualization or decay cannot dispense with such action. If you remove the cause, movement will stop. *Cessante causa cessat effectus.*[31]

If we are dealing with "natural" movement, this cause, this motor is the very nature of the body, its "form," which seeks to

[29] Cf. Kurt Riezler, *Physics and Reality* (New Haven, 1940).

[30] Local movement—locomotion—is only one, though a particularly important, kind of "motion" (*κίνησις*), motion in the realm of space, in contradistinction to alteration, motion in the realm of quality, and generation and decay, motion in the realm of being.

[31] Aristotle is perfectly right. No process of change or becoming can dispense with a cause. And if motion, in modern physics, persists by itself, it is because it is no longer a process.

bring it back to its place, and thus keeps the movement going. *Vice versa,* movement which is *contra naturam* requires throughout its duration the *continuous* action of an *external* mover conjoint to the moved. Remove the mover, and the movement will stop. Detach it from the moved, and the movement will equally stop. Aristotle, as we know well, does not admit action at a distance;[32] every transmission of movement implies according to him a contact. Therefore there are only two kinds of such transmission: pressure and traction. To move a body you have either to push or to pull it. There is no other means.

Aristotelian physics thus forms an admirable and perfectly coherent theory which, to tell the truth, has only one flaw (besides that of being false): that of being contradicted by everyday practice, by the practice of throwing. But a theoretician deserving the name does not allow himself to be troubled by an objection from common sense. If and when he encounters a "fact" that does not fit into his theory, he denies its existence. And if he cannot deny it, he explains it. And it is in the explanation of this everyday fact, the fact of throwing, a movement continuing in spite of the absence of a "mover," a fact apparently incompatible with his theory, that Aristotle gives us the measure of his genius. This answer consists in the explanation of the apparently motorless movement of the projectile by the reaction of the ambiant medium, the air, or the water.[33] The theory is a stroke of genius. Unfortunately (besides being false), from the point of view of common sense it is utterly impossible. No wonder therefore that the criticism of Aristotelian dynamics turns always to the same *questio disputata: a quo moveantur projecta?*

II

We shall come back in a moment to this *questio,* but we must first turn our attention to another detail of Aristotelian dynamics: the negation of any vacuum and of movement in a vacuum. In this dynamics, indeed, a vacuum does not enable movement to proceed more easily; on the contrary, it renders it utterly impossible; this for very profound reasons.

We have already said that in Aristotelian dynamics, every body is conceived as endowed with a tendency to find itself in its natural

[32] The body *tends* to its natural place, but it is not *attracted* by it.

[33] Cf. Aristotle, *Physics,* IV, 8, 215 a; VIII, 10, 267 a; *De Coelo,* III, 2, 301 b; É. Meyerson, *Identité et réalité,* 84.

place, and to come back to it when, and if, by violence it is moved away from it. This tendency explains its (natural) movement: a movement which brings it to its natural place by the shortest and the speediest way. It follows that every natural movement proceeds in a straight line, and that every body travels to its natural place as fast as possible; that is, as fast as its environment, which resists and opposes its movement, allows it to do. If therefore there were nothing to arrest it, if the surrounding medium did not oppose any resistance to its movement through it (as would be the case in a vacuum) the body would travel to ''its'' place with an infinite speed.[34] But such a movement would be instantaneous and this—with good reason—seems to Aristotle to be utterly impossible. The conclusion is obvious: no (natural) movement can possibly take place in the void. As for violent movement, that, for example, of throwing, movement in a vacuum would be equivalent to movement without a motor; it is obvious that the vacuum is not a physical medium and cannot receive, transmit and keep up a movement. Moreover, in a vacuum (as in the space of the Euclidian geometry) there are no privileged places or directions. In a vacuum there are not, and there cannot be, ''natural'' places. Therefore a body put into a vacuum would not know where to go, would not have any reason to move in one direction rather than in any other, and thus would not have any reason to move at all. *Vice versa,* once moved, it would have no more reason to stop here rather than there, and thus it would have no reason to stop at all.[35] Both of which are utterly absurd.

Aristotle is once more perfectly right. An empty space (the space of geometry) is utterly destructive of the conception of a cosmic order: in an empty space there are not only no natural places,[36] there are no *places* at all. The idea of a vacuum is not compatible with the interpretation of movement as change and as process—perhaps not even with that of the concrete movement of concrete ''real,'' perceptible, bodies: I mean the bodies of our common everyday experience. The vacuum is a *non-ens;*[37] and to place things in such a *non-ens* is absurd.[38] Geometrical bodies alone can be ''placed'' in a geometrical space.

[34] Cf. Aristotle, *Physics,* VII, 5, 249 b, 250 a; *De Coelo,* III, 2, 301 e.

[35] Cf. Aristotle, *Physics,* IV, 8, 214 b; 215 b.

[36] If one likes it better, one can say that in a vacuum all places are the natural places of every kind of body.

[37] Kant called empty space an *"Unding."*

[38] Such was, as we know, the opinion of Descartes; and of Spinoza.

The physicist investigates real things, the geometer reasons about abstractions. Therefore, contends Aristotle, nothing could be more dangerous than to mingle together geometry and physics, and to apply purely geometrical method and reasoning to the study of physical reality.

III

I have already mentioned that Aristotelian dynamics, in spite—or perhaps because—of its theoretical perfection, was burdened with an important draw-back; that of being utterly implausible and completely unbelievable and unacceptable to plain sound common sense, and obviously contradictory to the commonest everyday experience. No wonder therefore that it never enjoyed universal recognition, and that the critics and adversaries of the dynamics of Aristotle always opposed to it the common-sense fact of the persistence of movement separated from its original motor. Thus the classical examples of such movement, for instance the continuing rotation of the wheel, the flight of the arrow, the throwing of a stone, were persistently marshalled against it, beginning with Hipparchus and John Philoponos, through John Buridan and Nicole Oresme, down to Leonardo da Vinci, Benedetti and Galileo.[39]

I do not propose to analyze here the traditional arguments which since John Philoponos[40] have been repeated by the partisans of his dynamics. *Grosso modo* they can be classified into two

[39] For the history of the medieval criticism of Aristotle cf. the works cited above, n. 17, and B. Jansen, "Olivi, der älteste scholastische Vertreter des heutigen Bewegungsbegriffes," *Philosophisches Jahrbuch* (1920); K. Michalsky, "La physique nouvelle et les différents courants philosophiques au XIVe siècle," *Bulletin international de l'Académie polonaise des sciences et des lettres* (Cracovie, 1927); S. Moser, *Grundbegriffe der Naturphilosophie bei Wilhelm von Occam* (Innsbruck, 1932); E. Borchert, *Die Lehre von der Bewegung bei Nicolaus Oresme* (Münster, 1934); R. Marcolongo, "La Meccanica di Leonardo da Vinci," *Atti della reale accademia delle scienze fisiche e matematiche,* XIX (Napoli, 1933).

[40] On John Philoponos, who seems to be the real inventor of the theory of the *impetus,* cf. E. Wohlwill, "Ein Vorgänger Galileis im VI. Jahrhundert," *Physicalische Zeitschrift,* VII (1906), and P. Duhem, *Le Système du Monde,* I. The *Physics* of John Philoponos, not having been translated into Latin, remained inaccessible to the scholastics, who had at their disposal only the brief account given by Simplicius. But it was well known to the Arabs, and the Arabic tradition, directly and through the translation of Avicenna, seems to have influenced the "Parisian" school to a hitherto unsuspected degree. Cf. the very important article of S. Pines, "Études sur Awhad al-Zamān Abu'l Barakat al-Baghdadi," *Revue des Études Juives* (1938).

groups: a) the first arguments are material and stress the improbability of the assumption that a big and heavy body, a bullet, a revolving mill-stone, an arrow flying against the wind, could be moved by the reaction of the air; b) the others are formal and point out the contradiction involved in attributing to the air a double rôle, that of resistance and that of being a mover, as well as the illusory character of the whole theory which only shifts the problem from the body to the air and is, in fact, obliged to endow the air with the same ability to maintain its movement in spite of its separation from its external cause which it denies to other bodies. If so, they ask, why not assume that the mover transmits to the moved, or impresses it with, something which enables it to move—a something which is called δύναμις, *virtus motiva, virtus impressa, impetus, impetus impressus,* sometimes *forza* or even *motio,* and which is always thought of as some kind of power or force, which passes from the mover to the *mobile,* and which then carries on the movement, or better, which produces the movement as its cause.

It is obvious, as Duhem himself recognized, that we are back with common sense. The partisans of the *impetus* physics are thinking in terms of everyday experience. Is it not clear that we need an *effort,* a deployment and an expenditure of force, in order to move a body, for instance in order to push a carriage along its path, to throw a stone or to bend a bow? Is it not clear that it is this force which moves the body, or better, which makes it move?—that it is this force which the body receives from the mover that enables it to overcome resistance (like that of the air) and to strike at obstacles?

The medieval followers of *impetus* dynamics discuss at great length, and without success, the ontological status of *impetus.* They try to fit it into the Aristotelian classification, to interpret it as some kind of *form,* or as a kind of *habitus,* or as a kind of quality such as heat (like Hipparchus and Galileo). These discussions only show the confused, imaginative nature of the conception, which is a direct product or, if one may say so, a condensation, of common sense.

As such it is even more in accord than the Aristotelian view with the "facts"—real or imaginary—which form the experiential basis of medieval dynamics; and particularly with the well known "fact" that every projectile begins by increasing its speed and acquires the maximum of its velocity some time after its separation from the mover.[41] Everybody knows that in order to jump an obstacle one has to "make a take-off;" that a chariot which one pushes, or pulls, starts slowly and little by little increases its speed; it too takes off and gathers momentum; just as everybody—even a child throwing a ball—knows that in order to hit the goal hard he has to place himself at a certain distance from it, and not too near, in order to allow the ball to gather momentum. The physics of *impetus* is not at pains to explain this phenomenon; from its standpoint it is perfectly natural that *impetus* should require some time before it "takes hold" of the *mobile*—just as, for example, heat needs time to permeate a body.

The conception of movement underlying and supporting *impetus* physics is quite different from that of the Aristotelian view. Movement is no longer understood as a process of actualization.

[41] It is interesting to note that this absurd belief, shared and taught by Aristotle (*De Coelo,* II, 6), was so deeply rooted and so universally accepted that Descartes himself did not dare to deny it outright, and as so often with him preferred to explain it. In 1630 he writes to Mersenne (A. T., I, 110): "Je voudrais bien aussi sçavoir si vous n'avez point expérimenté si une pierre jettée avec une fronde, ou la bale d'un mousquet, ou un traist d'arbaleste, vont plus viste et ont plus de force au milieu de leur mouvement qu'ils n'en ont au commencement, et s'ils font plus d'effet. Car c'est là la créance du vulgaire, avec laquelle toutefois mes raisons ne s'accordent pas; et je trouve que les choses qui sont poussées et qui ne se meuvent pas d'elles mêmes, doivent avoir plus de force au commencement qu'incontinent après." In 1632 (A. T., I, 259) and once more in 1640 (A. T., II, 37 sq.) he explains to his friend what is true in this belief: *"In motu projectorum,* ie ne croie point que le Missile aille jamais moins vite au commencement qu'à la fin, à conter dès le premier moment qu'il cesse d'être poussé par la main ou la machine; mais je crois bien qu'un mousquet, n'estant éloigné que d'un pied et demi d'une muraille n'aura pas tant d'effet que s'il en était éloigné de quinze ou de vingt pas, à cause que la bale, en sortant du mousquet ne peut si aisement chasser l'air qui est entre lui et cette muraille et ainsi doit aller moins viste que si cette muraille estoit moins proche. Toutefois c'est à l'expérience de déterminer si cette différence est sensible et je doute fort de toutes celles que je n'ai pas faites moi-même." Descartes' friend, Beekmann, on the contrary, denies flatly the possibility of an acceleration of the projectile and writes (*Beekmann à Mersenne,* Apr. 30, 1630, cf. *Correspondance du Père Mersenne* [Paris, 1936], II, 437): "Funditores vero ac pueri omnes qui existimant remotiora fortius ferire quam eadem propinquiora, certo certius falluntur." Yet he admits that there must be something true in this belief and tries to explain: "Non dixeram plenitudinem nimiam aeris impedire effectum tormentorii globi, sed pulverem pyrium extra bombardam jam existentem forsitan adhuc rarefieri, ideoque fieri posse ut globus tormentarius extra bombardam nova vi (simili tandem) propulsus velocitate aliquamdiu cresceret."

Yet it is still a change, and as such it must be explained by the action of a definite force or cause. *Impetus* is just that immanent cause which produces the movement, which is *converso modo* the effect produced by it. Thus the *impetus impressus produces* the movement; it *moves* the body. But at the same time it plays another very important rôle: it overcomes the resistance opposed by the medium to the movement.

Owing to the confused and ambiguous character of the *impetus* conception, it is rather natural that the two aspects and rôles should merge together, and that some of the partisans of the *impetus* dynamics should come to the conclusion that, at least in some special cases such as the circular movement of the heavenly spheres, or, more generally, the rolling movement of a circular body on a level plane, or even more generally in all the cases where there is no external resistance to movement, such as would be the case in a *vacuum*, the *impetus* does not weaken but remains "immortal." This seems to be a close approach to the law of inertia, and it is therefore of particular interest and importance to note that Galileo himself, who in his *De Motu* gives us one of the best expositions of *impetus* dynamics, resolutely denies the possibility of such an assumption, and asserts most vigorously the essentially perishable nature of *impetus*.

Galileo is obviously perfectly right. If movement is understood as the effect of *impetus* considered as its immanent—and not natural—cause, it is unthinkable and absurd not to admit that the cause or force which produces it must necessarily spend and finally exhaust itself in this production. It can never remain unchanged for two consecutive moments, and therefore the movement which it produces must necessarily slow down and come to an end.[42] Thus it is a very important lesson that we learn from the young Galileo. He teaches us that *impetus* physics, though compatible with movement in a *vacuum*, is like that of Aristotle *incompatible* with the principle of inertia. And this is not the only lesson that Galileo teaches with regard to *impetus* physics. The second is at least as valuable as the first. It runs that, like that of Aristotle, the dynamics of *impetus* is incompatible with mathematical treatment. It leads nowhere. It is a blind alley.

Impetus physics, during the thousand years that separate John Philoponos from Benedetti, made very little progress. But in the

[42] Cf. Galileo Galilei, *De Motu, Opere,* Ed. Naz., I, 314 sq.

latter's works, and even more clearly, more consistently and consciously, in those of the young Galileo, we find—under the obvious and unmistakable influence of the "suprahuman Archimedes"[43] a determined attempt to apply to this physics the principles of "mathematical philosophy."[44]

Nothing is more instructive than the study of this attempt—or, more exactly, of these attempts—and of their failure. They show us that it is impossible to mathematize, i.e., to transform into an exact, mathematical concept, the rude, vague and confused conception of *impetus*. In order to build up a mathematical physics following the lines of the statics of Archimedes, this conception had to be dropped altogether.[45] A new and original concept of motion had to be formed and developed. It is this new concept that we owe to Galileo.

IV

We are too well acquainted with, or rather too well accustomed to, the principles and concepts of modern mechanics, so well that it is almost impossible for us to see the difficulties which had to be overcome for their establishment. They seem to us so simple, so natural, that we do not notice the paradoxes they imply and contain. Yet the mere fact that the greatest and mightiest minds of mankind—Galileo, Descartes—had to struggle in order to make them theirs, is in itself sufficient to indicate that these clear and simple notions—the notion of movement or that of space—are not so clear and simple as they seem to be. Or they are clear and simple only from a certain point of view, only as part of a certain set of concepts and axioms, apart from which they are not simple at all. Or, perhaps, they are too clear and too simple: so clear and so simple that, like all prime notions, they are very difficult to grasp.

Movement, space—let us try to forget for a while all we have learnt at school; let us try to think out what they mean in mechanics. Let us try to place ourselves in the situation of a contemporary of Galileo, a man accustomed to the concepts of Aris-

[43] Galileo Galilei, *De Motu,* 300.

[44] J. B. Benedetti, *Diversarum speculationum mathematicarum liber* (Taurini, 1585), 168.

[45] The persistence of the terminology—the word *impetus* is used by Galileo and his pupils and even by Newton—must not prevent us from recognizing the disappearance of the idea.

totelian physics which *he* learnt at *his* school, and who encounters for the first time the modern concept of motion. What is it? In fact something pretty strange. It is something which in no way affects the body which is endowed with it: to be in motion or to be at rest does not make any difference for, nor any change in, the body in motion or at rest. The body, as such, is utterly and absolutely indifferent to both.[46] Therefore, we are not able to ascribe motion to a determined body considered in itself. A body is in motion only in relation to some other body which we assume to be at rest. All motion is relative. And therefore we may ascribe it to the one or to the other of the two bodies, *ad libitum*.[47]

Thus motion seems to be a relation. But at the same time it is a *state,* just as rest is another *state,* utterly and absolutely opposed to the former; besides which they are both *persistent states*.[48] The famous first law of motion, the law of inertia, teaches us that a body left to itself persists eternally in its state of motion or of rest, and that we must apply a force in order to change a state of motion to a state of rest, and *vice versa*.[49] Yet not every kind of motion is thus endowed with an eternal being, but only uniform movement in a straight line. Modern physics affirms, as well we know, that a body once set in motion conserves eternally its direction and speed, provided of course it is not subject to the action of any external force.[50] Moreover, to the objection of the Aristotelian that though as a matter of fact he is acquainted with eternal motion, the eternal circular motion of the heavenly spheres, he has never yet encountered a persistent rectilinear one, modern physics replies: of course! rectilinear, uniform motion is utterly impossible, and can take place only in a vacuum.

Let us think it over, and perhaps we will not be too harsh on the Aristotelian who felt himself unable to grasp and to accept this

[46] In the Aristotelian physics, motion is a process of change and always affects the body in motion.

[47] A given body, therefore, can be endowed with any number of different motions, which do not interfere with each other. In the Aristotelian as well as in the *impetus* physics every motion interferes with every other and sometimes even prevents it from taking place.

[48] Motion and rest are thus placed on the same ontological level, and therefore persistence of *motion* becomes just as self-evident and without need of explanation as persistence of *rest* had previously been.

[49] In modern terms: in the Aristotelian and *impetus* dynamics, force produces motion; in modern dynamics, force produces acceleration.

[50] This implies necessarily the infinity of the Universe.

unheard-of notion, the notion of a persistent, substantial relation-state, the concept of something which to him seemed just as abstruse, and just as impossible, as the ill-fated substantial forms of the scholastics appear to us. No wonder that the Aristotelian felt himself astonished and bewildered by this amazing attempt to explain the real by the impossible—or, which is the same thing, to explain real being by mathematical being, because, as I have mentioned already, these bodies moving in straight lines in infinite empty space are not *real* bodies moving in *real* space, but *mathematical* bodies moving in *mathematical* space.

Once more, we are so accustomed to mathematical science, to mathematical physics, that we no longer feel the strangeness of a mathematical approach to Being, the paradoxical daring of Galileo's utterance that the book of Nature is written in geometrical characters.[51] For us it is a foregone conclusion. But not for the contemporaries of Galileo. Therefore it is the right of mathematical science, of the mathematical explanation of Nature, in opposition to the non-mathematical one of common sense and of Aristotelian physics, much more than the opposition between two astronomical systems, that forms the real subject of the *Dialogue on the Two Greatest Systems of the World.* As a matter of fact the *Dialogue,* as I believe I have shown in my ill-fated volume, is not so much a book on *science* in our meaning of the term as a book on philosophy—or to be quite correct and to employ a disused but time-honored expression, a book on *natural philosophy*—for the simple reason that the solution of the astronomical problem depends on the constitution of a new Physics; which in turn implies the solution of the *philosophical* question of the rôle played by mathematics in the constitution of the science of Nature.

The rôle and the place of mathematics in science is not in fact a very new problem. Quite the contrary: for more than two thousand years it has formed the object of philosophical meditation, inquiry and discussion. And Galileo is perfectly aware of it. No wonder! Even as a young boy, a student in the University of Pisa,

[51] G. Galilei, *Il Saggiatore* (*Opere,* VI, 232): "La filosofia è scritta in questo grandissimo libro, che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola." Cf. *Letter to Liceti* of Jan. 11, 1641 (*Opere,* XVIII, 293).

he could have learned from the lectures of his master, Francesco Buonamici, that the "question" about the rôle and the nature of mathematics constitutes the principal subject of opposition between Aristotle and Plato.[52] And some years later when he came back to Pisa, this time a professor himself, he could have learned from his friend and colleague, Jacopo Mazzoni, author of a book on Plato and Aristotle, that "there is no other question which has given place to more noble and beautiful speculations . . . than the question whether the use of mathematics in physical science as an instrument of proof and a middle term of demonstration, is opportune or not; in other words, whether it brings us some profit, or on the contrary is dangerous and harmful." "It is well known," says Mazzoni, "that Plato believed that mathematics was quite particularly appropriate for physical investigations, which was the reason

[52] The enormous compilation of Buonamici (1011 pages *in folio*) is an invaluable source-book for the study of medieval theories of motion. Though frequently *mentioned* by historians of Galileo it has never been utilized by them. Buonamici's book is very rare. I allow myself therefore to quote it at some length: Francisci Bonamici, Florentini, e primo loco philosophiam ordinariam in Almo Gymnasio Pisano profitentis, *De Motu, libri X, quibus generalia naturalis philosophiae principia summo studio collecta continentur* (Florentiae, 1591), lib. X, cap. XI. *Jurene mathematicae ex ordine scientiarum expurgantur,* p. 56: . . . "Itaque veluti ministri sunt mathematicae, nec honore dignae et habitae προπαιδεία, id est apparatus quidam ad alias disciplinas. Ob eamque potissime caussam, quod de bono mentionem facere non videntur. Etenim omne bonum est finis, is vero cuiusdam actus est. Omnis vero actus est cum motu. Mathematicae autem motum non respiciunt. Haec nostri addunt. Omnem scientiam ex propriis effici: propria vero sunt necessaria quae alicui [?] quatenus ipsum et per se insunt. Atqui talia principia mathematicae non habent. . . . Nullum caussae genus accipit . . . propterea quod omnes caussae definiuntur per motum: efficiens enim est principium motus, finis cuius gratia motus est, forma et materia sunt naturae; et motus igitur principia sint necesse est. At vero mathematica sunt immobilia. Et nullum igitur ibi caussae genus existit." *Ibid.,* lib. I, p. 54 "Mathematicae cum ex notis nobis et natura simul efficiant id quod cupiunt, sed caeteris demonstrationis perspicuitate praeponentur, nam vis rerum quas ipsae tractant non est admodum nobilis; quippe quod sunt accidentia, id est habeant rationem substantiae quatenus subiicitur et determinatur quanto; eaque considerentur longe secus atque in natura existant. Attamen nonnullarum rerum ingenium tale esse comperimus ut ad certam materiam sese non applicent, neque motum consequantur, quia tamen in natura quicquid est, cum motu existit; opus est abstractione cuius beneficio quantum motu non comprehenso in eo munere contemplamur; et cum talis sit earum natura nihil absurdi exoritur. Quod item confirmatur, quod mens in omni habitu verum dicit; atqui verum est ex eo, quod res ita est. Huc accedit quod Aristoteles distinguit scientias non ex ratione notionum sed entium."

why he himself had many times recourse to it for the explanation of physical mysteries. But Aristotle held a quite different view and he explained the errors of Plato by his too great attachment to mathematics.''[53]

One sees that for the scientific and philosophical consciousness of the time—Buonamici and Mazzoni are only giving expression to the *communis opinio*—the opposition, or rather the dividing line, between the Aristotelian and the Platonist is perfectly clear. If you claim for mathematics a superior status, if more than that you attribute to it a real value and a commanding position in Physics, you are a Platonist. If on the contrary you see in mathematics an abstract science, which is therefore of a lesser value than those—physics and metaphysics—which deal with real being; if in particular you pretend that physics needs no other basis than experience and must be built directly on perception, that mathematics has to content itself with the secondary and subsidiary rôle of a mere auxiliary, you are an Aristotelian.

What is in question in this discussion is not certainty—no Aristotelian has ever doubted the certainty of geometrical propositions or demonstrations—but Being; not even the use of mathematics in physical science—no Aristotelian has ever denied our right to measure what is measurable and to count what is numerable—but the structure of science, and therefore the structure of Being.

[53] Jacobi Mazzoni, Caesenatis, in Almo Gymnasio Pisano Aristotelem ordinarie Platonem vero extra ordinem profitentis, *In Universam Platonis et Aristotelis Philosophiam Praeludia, sive de comparatione Platonis et Aristotelis* (Venetiis, 1597), 187 sq.: *Disputatur utrum usus mathematicarum in Physica utilitatem vel detrimentum afferat, et in hoc Platonis et Aristotelis comparatio.* Non est enim inter Platonem et Aristotelem quaestio, seu differentia, quae tot pulchris, et nobilissimis speculationibus scateat, ut cum ista, ne in minima quidem parte comparari possit. Est autem differentia, utrum usus mathematicarum in scientia Physica tanquam ratio probandi et medius terminus demonstrationum sit opportunus, vel inopportunus, id est, an utilitatem aliquam afferat, vel potius detrimentum et damnum. Credidit Plato Mathematicas ad speculationes physicas apprime esse accommodatas. Quapropter passim eas adhibet in reserandis mysteriis physicis. At Aristoteles omnino secus sentire videtur, erroresque Platonis adscribet amori Mathematicarum. . . . Sed si quis voluerit hanc rem diligentius considerare, forsan, et Platonis defensionem inveniet, videbit Aristotelem in nonnullos errorum scopulos impegisse, quod quibusdam in locis Mathematicas demonstrationes proprio consilio valde consentaneas, aut non intellexerit, aut certe non adhibuerit. Utramque conclusionem, quarum prima ad Platonis tutelam attinet, secunda errores Aristotelis ob Mathematicas male rejectas profitetur, brevissime demonstrabo."

These are the discussions to which Galileo alludes continuously in the course of his *Dialogue.* Thus at the very beginning Simplicio, the Aristotelian, points out that "concerning natural things we need not always seek the necessity of mathematical demonstrations."[54] To which Sagredo, who allows himself the pleasure of misunderstanding Simplicio, replies: "Of course, when you cannot reach it. But, if you can, why not?" Of course. If it is possible in questions pertaining to natural things to achieve a demonstration possessing a mathematical necessity, why shouldn't we try to do it? But is it possible? That is precisely the problem, and Galileo, in the margin of the book, sums up the discussion and formulates the real meaning of the Aristotelian: "In natural demonstrations," says he, "one must not seek mathematical exactitude."

One must not. Why? Because it is impossible. Because the nature of physical being is qualitative and vague. It does not conform to the rigidity and the precision of mathematical concepts. It is always "more or less." Therefore, as the Aristotelian will explain to us later, philosophy, that is the science of the real, does not need to look at details, nor need it have recourse to numerical determinations in formulating its theories of motion; all that it has to do is to develop its chief categories (natural, violent, rectilinear, circular) and to describe its general qualitative and abstract features.[55]

The modern reader is probably far from being convinced. He finds it difficult to admit that "philosophy" had to content itself with abstract and vague generalization and not try to establish precise and concrete universal laws. The modern reader does not know the real reason of this necessity, but Galileo's contemporaries knew it quite well. They knew that quality, as well as form, being non-mathematical by nature, cannot be treated in terms of mathematics. Physics is not applied geometry. Terrestrial matter can never exhibit exact mathematical figures; the "forms" never "inform" it completely and perfectly. There always remains a gap. In the skies, of course, it is different; and therefore mathematical astronomy is possible. But astronomy is not physics. To have missed that point is precisely the error of Plato and of those who follow Plato. It is useless to attempt to build up a mathematical

[54] Cf. Galileo Galilei, *Dialogo sopra i due Massimi Sistemi del Mondo, Opere,* Ed. Naz., VII, 38; cf. 256.

[55] Cf. *Dialogo,* 242.

philosophy of nature. The enterprise is doomed even before it starts. It does not lead us to truth but to error.

"All these mathematical subtleties," explains Simplicio, "are true in *abstracto*. But applied to sensible and physical matter, they do not work."[56] In real nature there are no circles, no triangles, no straight lines. Therefore it is useless to learn the language of mathematical figures: the book of Nature, in spite of Galileo and Plato, is not written in them. In fact, it is not only useless, it is dangerous: the more a mind is accustomed to the precision and to the rigidity of geometrical thought, the less it will be able to grasp the mobile, changing, qualitatively determined variety of Being.

This attitude of the Aristotelian is very far from being ridiculous.[57] To me, at least, it seems perfectly sensible. You cannot establish a mathematical theory of quality, objects Aristotle to Plato; not even one of motion. There is no motion in numbers. But *ignorato motu ignoratur natura.* And the Aristotelian of Galileo's time could add that the greatest of the Platonists, the *divus* Archimedes himself,[58] was never able to establish more than a statics. Not a dynamics. A theory of rest. Not one of motion.

The Aristotelian was perfectly right. It is impossible to furnish a mathematical deduction of quality. And well we know that Galileo, like Descartes somewhat later, and for just the same reason, was forced to drop the notion of quality, to declare it subjective, to ban it from the realm of nature.[59] This at the same time implies that he was obliged to drop sense-perception as the source of knowledge and to proclaim that intellectual, and even *a priori* knowledge, is our sole and only means of apprehending the essence of the real.

As for dynamics, and the laws of motion—the *posse* is only to be proved by the *esse;* in order to show that it is possible to establish mathematical laws of nature, you have to do it. There is no other way and Galileo is perfectly conscious of it. It is therefore by giving mathematical solutions to concrete physical problems—the problem of falling bodies, the problem of projectile motion—

[56] *Ibid.,* 229, 423.

[57] As we know, it was shared by Pascal, and even by Leibniz.

[58] It is perhaps worth mentioning that for all the doxographic tradition, Archimedes is a *philosophus platonicus.*

[59] Cf. E. A. Burtt, *The Metaphysical Foundations of Modern Physical Science* (London and New York, 1925).

that he leads Simplicio to the confession "that to want to study natural problems without mathematics is to attempt something that cannot be done."

It seems to me that we are now able to understand the meaning of this significant text of Cavalieri, who in 1630 writes in his *Specchio Ustorio:* "How much is added by the knowledge of the mathematical sciences, which the famous schools of Pythagoreans and Platonists considered supremely necessary for the comprehension of physical things, I hope will shortly become clear with the publication of the new science of movement promised by this marvellous Assayer of Nature, Galileo Galilei."[60]

And we understand too the pride of Galileo the Platonist, who in his *Discourses and Demonstrations* announces that "about a most ancient subject he will promote a quite new science," and will prove something that nobody has proven till then, namely that the movement of falling bodies is subjected to the law of numbers.[61] Movement governed by numbers; the Aristotelian objection had at last met its refutation.

It is obvious that for the disciples of Galileo just as for his contemporaries and elders mathematicism means Platonism. Therefore when Torricelli tells us "that among the liberal disciplines geometry *alone* exercises and sharpens the mind and renders it able to be an ornament of the City in time of peace and to defend it in time of war," and that "*caeteris paribus,* a mind trained in geometrical gymnastics is endowed with a quite particular and *virile* strength,"[62] not only does he show himself an authentic

[60] Bonaventura Cavalieri, *Lo Specchio Ustorio overo trattato Delle Settioni Coniche e alcuni loro mirabili effetti intorno al Lume* etc. (Bologna, 1632), 152 sq.: "Ma quanto vi aggiunga la cognitione delle scienze Matematiche, giudicate da quelle famosissime scuole de' Pithogorici et de' 'Platonici,' sommamente necessarie per intender le cose Fisiche, spero in breve sarà manifesto, per la nuova dottrina del moto promessaci dall'esquisitissimo Saggiatore della Natura, dico dal Sig. Galileo Galilei, ne' suoi Dialoghi. . . ."

[61] Galileo Galileo, *Discorsi e dimostrazioni mathematiche intorno a due nuove scienze* (*Opere,* Ed. Naz., VIII, 190): "nullus enim, quod sciam, demonstravit, spatia a mobile descendente ex quiete peracta in temporibus aequalibus, eam inter se retinere rationem, quam habent numeri impares ab unitate consequentes."

[62] Evangelista Torricelli, *Opera Geometrica* (Florentiae, 1644), II, 7: "Sola enim Geometria inter liberales disciplinas acriter exacuit ingenium, idoneumque reddit ad civitates adornandas in pace et in bello defendendas: caeteris enim paribus, ingenium quod exercitatum sit in Geometrica palestra, peculiare quoddam et virile robur habere solet: praestabitque semper et antecellet, circa studia Architecturae, rei bellicae, nauticaeque, etc."

disciple of Plato, he acknowledges and proclaims himself to be one. And in doing it he remains a faithful disciple of his master Galileo, who in his *Response to the Philosophical Exercitations* of Antonio Rocco addresses himself to the latter, asking him to judge for himself the value of the two rival methods, i.e., the purely physical and empirical method and the mathematical one, adding: "and decide at the same time who reasoned better, Plato, who said that without mathematics one could not learn philosophy, or Aristotle, who reproached this same Plato for having too much studied Geometry."[63]

I have just called Galileo a Platonist. And I believe that nobody will doubt that he is one.[64] Moreover, he says so himself. In the very first pages of the *Dialogue* Simplicio makes the remark that Galileo, being a mathematician, is probably sympathetic to the numerical speculations of the Pythagoreans. This enables Galileo to declare that he deems them perfectly meaningless, and to say at the same time: "I know perfectly well that the Pythagoreans had the highest esteem for the science of number and that Plato himself admired the human intellect and believed that it participates in divinity solely because it is able to understand the nature of numbers. And I myself am well inclined to make the same judgment."[65]

[63] Galileo Galilei, *Esercitazioni filosofiche di Antonio Rocco, Opere,* Ed. Naz., VII, 744.

[64] The Plationism of Galileo Galilei has been more or less clearly recognized by certain modern historians of science and philosophy. Thus the author of the German translation of the *Dialogo* notes the Platonic influence (the doctrine of anamnesis) on the very form of the book (cf. G. Galilei, *Dialog über die beiden hauptsächlichsten Weltsysteme,* aus dem italienischen übersetzt und erläutert von E. Strauss [Leipzig, 1891], p. XLIX); E. Cassirer (*Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit,* 2 ed. [Berlin, 1911], I, 389 sq.) insists upon the Platonism of Galileo's ideal of knowledge; L. Olschki (*Galileo und seine Zeit,* Leipzig, 1927) speaks about the "Platonic vision of Nature" of Galileo, etc. It is E. A. Burtt, *The Metaphysical Foundations of Modern Physical Science* (New York, 1925), who seems to me to have given the best account of the metaphysical substructure (Platonic mathematicism) of modern science. Unfortunately Burtt failed to recognize the existence of *two* (and not one) Platonic traditions, that of mystical arithmology, and that of mathematical science. The same error, which in the case of Burtt was a venial sin, was made by his critic, E. W. Strong, *Procedures and Metaphysics* (Berkeley, Cal., 1936), and in this case it was a mortal one.—On the distinction between the two Platonisms cf. L. Brunschvicg, *Les Étapes de la philosophie mathématique* (Paris, 1922), 69 sq., and *Le Progrès de la conscience dans la philosophie occidentale* (Paris, 1937), 37 sq.

[65] *Dialogo,* 35.

How could he be of a different opinion, he who believed that in mathematical knowledge the human mind attains the very perfection of the divine understanding? Does he not say that "*extensive,* that is in respect of the multiplicity of things to be known, which is infinite, the human mind is as nothing (even if it understood a thousand propositions, because a thousand compared with infinity is like zero): but taking the understanding *intensive,* in so far as this term means to grasp intensely, that is, perfectly a given proposition, I say that the human mind understands some propositions as perfectly and has of them as absolute certainty as Nature herself can have; and of that kind are the pure mathematical sciences, that is, geometry and arithmetic, of which the divine intellect knows of course infinitely more propositions, for the simple reason that it knows them all; but as for those few understood by the human intellect, I believe that our knowledge equals the divine in objective certainty, because it succeeds in understanding their necessity, beyond which it does not seem that there can exist a greater certainty."[66]

Galileo could have added that the human understanding is so excellent a work of God that *ab initio* it is in possession of these clear and simple ideas of which the very simplicity is a guarantee of truth, and that it has only to turn to itself in order to find in its "memory" the true foundations of science and knowledge, the alphabet, i.e., the elements, of the language—the mathematical language—spoken by the Nature God has created. There is to be found the true foundation of a *real* science, a science of the *real* world—not of a science endowed with a purely formal truth, the intrinsic truth of mathematical reasoning and deduction, a truth which would not be affected by the non-existence in Nature of the objects studied by it: it is obvious that Galileo would no more than Descartes ever rest content with such an *Ersatz* for real science and knowledge.

It is of this science, the true "philosophic" knowledge which is knowledge of the very essence of Being, that Galileo proclaims: "And I, I say to you that if one does not know the truth by himself, it is impossible for anyone else to give him that knowledge. It is indeed possible to teach those things that are neither true nor false; but the true, by which I mean necessary things, that is, those for which it is impossible to be otherwise, every average mind

[66] *Dialogo,* 128 sq.

either knows by itself, or it is impossible for it ever to learn them."[67] Assuredly. A Platonist cannot be of a different opinion because for him to know is nothing else than to understand.

The allusions to Plato so numerous in the works of Galileo, and the repeated mention of the Socratic maieutics and of the doctrine of reminiscence, are not superficial ornaments born from his desire to conform to the literary mode inherited from the concern of Renaissance thought with Plato. Nor are they meant to gain for the new science the sympathy of the "common reader," tired and disgusted by the aridity of Aristotelian scholastics; nor to cloak himself against Aristotle in the authority of his master and rival, Plato. Quite the contrary: they are perfectly serious, and must be taken at their face value. Thus, that no one might have the slightest doubt concerning his philosophical standpoint, Galileo insists:[68] "SALVIATI: The solution of the question under discussion implies the knowledge of certain truths that are just as well known to you as to me. But, as you do not remember them, you do not see that solution. In this way, without teaching you, because you know them already, but only by recalling them to you, I shall make you solve the problem yourself."

"SIMPLICIO: Several times I have been struck by your manner of reasoning, which makes me think that you incline to the opinion of Plato that *nostrum scire sit quoddam reminisci;* pray, free me from this doubt and tell me your own view."

"SALV: What I think of this opinion of Plato I can explain by words, and also by facts. In the arguments so far advanced I have already more than once declared myself by fact. Now I will apply the same method in the inquiry we have in hand, an inquiry which may serve as an example to help you more easily to understand my ideas concerning the acquisition of science. . . ."

The inquiry "we have in hand" is nothing else than the deduction of the fundamental propositions of mechanics. We are informed that Galileo judges he has done more than merely declare himself a follower and a partisan of Platonic epistemology. In addition, by applying it, by discovering the true laws of physics, by letting them be deduced by Sagredo and Simplicio, that is, *by the reader* himself, by *us,* he believes he has demonstrated the truth of Platonism "by fact." The *Dialogue* and the *Discourses* give

[67] *Dialogo,* 183.

[68] *Dialogo,* 217.

us the history of an intellectual experiment—of a conclusive experiment, because it ends with the wistful confession of the Aristotelian Simplicio, acknowledging the necessity of the study of mathematics, and regretting that he himself had not learned them in his youth.

The *Dialogue* and the *Discourses* tell us the history of the discovery, or better still, of the rediscovery of the language spoken by Nature. They explain to us the manner of questioning her, i.e., the theory of that scientific experimentation in which the formulation of postulates and the deduction of their implications precedes and guides the recourse to observation. This too, at least for Galileo, is a proof "by fact." The new science is for him an experimental proof of Platonism.

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